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Quantum Corrections for a Bardeen Regular Black Hole

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In this paper, we study the quantum corrections to the thermodynamical quantities (temperature and entropy) for a Bardeen charged regular black hole by using a quantum tunneling approach over semiclassical approximations. Taking into account the quantum effects, the semiclassical Bekenstein-Hawking temperature and the area law are obtained, which are then used in the first law of thermodynamics to evaluate corrections to these quantities. It is interesting to mention here that these corrections reduce to the corresponding corrections for the Schwarzschild black hole when the charge $e = 0$.

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I. INTRODUCTION

According to General Relativity, a black hole (BH) is a region of space from which nothing, including light, can escape. Quantum mechanics suggests that BHs are not black, but possess temperature and emit radiation (energy) continuously, called the Hawking radiation [1]. Bekenstein [2] predicted that BHs should have finite, non-zero temperature and entropy. Many attempts [3, 4] have been made to address the quantum mechanics of a scalar particle to obtain BH radiation. The entropy of the Kerr-Newmann and the de

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Sitter spacetimes has been found to be always equal to one quarter the area of the event horizon in fundamental units. However, tunneling provides the best way to visualize the source of radiation as compared to the approach mentioned above. Quantum tunneling refers to the phenomenon of particle's ability to penetrate energy barriers within electronic structures. This technique is based on electron-positron pair production, which requires an electric field.

It is a well-known fact that as a particle crosses the event horizon, there is a change of energy. For tunneling, it is assumed that the particles follow trajectories that are not allowed classically [5-7]. The evaporation of a BH is related to the emission of quantum particles as Hawking radiation allows BHs to lose mass. Black holes that lose more matter than they gain through other means are expected to dissipate, shrink and ultimately vanish. This process causes a change in the characteristic parameters (mass, angular momentum and charge), which alters the thermodynamics of BH. The motion of the particle may be in the form of outgoing or ingoing radial null geodesics. For the outgoing geodesics, the particle must be imaginary while for the ingoing geodesics, it is assumed to be real. This is due to the fact that only a real particle that has a speed less than or equal to the speed of light can exist inside the event horizon.

In order to calculate the imaginary part of the action, Parikh and his collaborator [5] used a procedure based on the purely radial null geodesic method. Srinivasan and Padmanabhan [6] introduced another technique based on the Hamilton-Jacobi method. Planck's scale correction in the Parikh-Wilczek tunneling framework was studied by many people [8-10]. Jiang *et al.* [8] investigated Hawking radiation as massless charged particles tunneling across the event horizon of Kerr and Kerr-Newmann BHs. Xu and Chen [9] evaluated the total flux of Hawking radiation for the Kerr-(anti) de-Sitter BHs. Liu and Zhu [10] found the emission rate of massless particles tunneling through the corrected horizon. Banerjee and Majhi [11] discussed the role of chirality by connecting the anomaly and the tunneling formalisms for Hawking radiation from BH. The same authors [12] also obtained the Hawking black body spectrum with the appropriate temperature for a BH. Majhi [13] derived the Hawking radiation and performed BH thermodynamic spectroscopy; i.e., the spectrum of entropy and area were obtained by using a density matrix technique. Banerjee *et al.* [14] found the entropy spectrum of a BH by using the tunneling criterion.

The quantum geometry of the BH horizon has been studied using loop

quantum gravity. Loop quantization reproduces the result of BH entropy originally discovered by Bekenstein and Hawking. Further, it leads to the computation of a quantum gravity correction to the entropy and the radiation of a BH.

Banerjee and Majhi [7] analyzed Hawking radiation as tunneling by using the Hamilton-Jacobi method. They computed quantum corrections to the Hawking temperature and the Bekenstein-Hawking area law of the Schwarzschild, anti-de Sitter Schwarzschild and Kerr BHs. Banerjee and Modak [15] gave a new conceptually simple approach to obtain the entropy for any stationary BH. They determined the semiclassical BH entropy for the most general Kerr-Newmann spacetime. Following the same approach, Akbar and Saifullah [16] studied quantum corrections to the entropy and the horizon area for the Kerr-Newmann, charged rotating BTZ and Einstein-Maxwell dilaton-axion BHs. Recently, Larrañaga [17] extended this type of work for a charged BH of string theory and for the Kerr-Sen BH. In this paper, we work out the temperature and the entropy corrections for a Bardeen regular BH. This is a generalization of the entropy correction for the Schwarzschild BH [7].

Banerjee and Majhi [18] computed corrections to the Hawking temperature and Bekenstein-Hawking entropy for the Schwarzschild BH by using the tunneling formalism based on the quantum WKB approximation. Majhi [19] analyzed the Hawking radiation as tunneling of a Dirac particle through an event horizon by applying the Hamilton-Jacobi method beyond the semiclassical approximation. Majhi and Samanta [20] investigated the tunneling of a photon and a gravitino through an event horizon by applying the Hamilton-Jacobi method beyond the semiclassical approximation. Banerjee *et al.* [21] discussed the quantum gravitational correction to the Hawking temperature from the Lemaitre-Tolman-Bondi model in a semiclassical approximation. They obtained the standard expression for the Hawking temperature and its first quantum gravitational correction.

The plan of this paper is as follows: Section **II** describes basic equations for the corrections using the Hamilton-Jacobi method, the first law of thermodynamics and the exactness condition. In Section **III**, we evaluate semiclassical thermodynamical quantities for a Bardeen regular BH. These quantities evaluate corrections to the temperature and the entropy. Finally, in the last section, we present the outlook of the paper.

II. REVIEW

In this section, we review some basic material used to evaluate the corrections to the entropy and the temperature. We use the Hamilton-Jacobi method [7] to compute the imaginary part of the action outside the semiclassical approximation by admitting all possible quantum corrections. Here, we write the expression for the quantum correction of a general function $S(r,t)$. We expand this function in terms of a series in powers of $\hbar$, *i.e.*,

$$S(r,t) = S_0(r,t) + \hbar S_1(r,t) + \hbar^2 S_2(r,t) + ... = S_0(r,t) + \sum_i \hbar^i S_i(r,t), \quad (1)$$

where $i = 1, 2, 3, ...$, S_0 is the semiclassical value and the terms involving $\hbar$ and its higher powers are considered as correction terms. The dimension of S_i (proportional to S_0) is $\hbar$ while the proportionality constants is $(\hbar^i)^{-1}$. The Planck's constant, $\hbar$, is of the order of the square of the Planck's mass (for $G = c = 1$). Dimensional analysis provides the proportionality constants with dimensions of m^{-2i}, where m is the mass of the BH. The most general expression for S in Eq. (1) can be written as

$$S(r,t) = S_0(r,t) + \sum_i \alpha_i \frac{\hbar^i}{m^{2i}} S_0(r,t) = S_0(r,t)\left(1 + \sum_i \alpha_i \frac{\hbar^i}{m^{2i}}\right), \quad (2)$$

where $S_0(r,t)$ is the semiclassical entropy and the remaining terms represent quantum corrections.

The modified form of the temperature of the BH can be written as

$$T = T_H\left(1 + \sum_i \alpha_i \frac{\hbar^i}{m^{2i}}\right)^{-1}, \quad (3)$$

where T_H is the standard semiclassical Hawking temperature and the terms with α_i are corrections due to quantum effects [7]. The dimensionless parameter α_i corresponds to the higher order loop corrections to the surface gravity $\kappa = 2\pi T$ of the BH. The corrected form of surface gravity is given by

$$\kappa = \kappa_0\left(1 + \sum_i \alpha_i \frac{\hbar^i}{m^{2i}}\right)^{-1}, \quad (4)$$

where $\kappa_0 = 2\pi T_H$ is the standard semiclassical surface gravity. If we consider α_i in terms of a single dimensionless parameter β such that $\alpha_i = \beta^i$, then we

get

$$1+\sum_i \alpha_i \frac{\hbar^i}{m^{2i}} = 1+\left(\frac{\beta\hbar}{m^2}+\frac{\beta^2\hbar^2}{m^4}+\frac{\beta^3\hbar^3}{m^6}+\frac{\beta^4\hbar^4}{m^8}+...\right) = \left(1-\frac{\beta\hbar}{m^2}\right)^{-1}. \quad (5)$$

Consequently, Eq. (4) reduces to

$$\kappa = \kappa_0\left(1-\frac{\beta\hbar}{m^2}\right); \quad (6)$$

hence, Eq. (3) can be written as

$$T = T_H\left(1-\frac{\beta\hbar}{m^2}\right). \quad (7)$$

Now we follow Ref.15 to evaluate an expression of the entropy for a charged regular BH. The first law of thermodynamics for two parameters m and e, *i.e.*, the mass and charge of the BH, respectively, can be written as

$$dm = TdS + \Phi de, \quad (8)$$

where T, S and Φ are the temperature, entropy and electrostatic potential of the BH, respectively. Equation (8) can be written as

$$dS(m,e) = \frac{1}{T}dm - \frac{\Phi}{T}de. \quad (9)$$

A differential of a function $f = f(x,y)$ is given by

$$df(x,y) = A(x,y)dx + B(x,y)dy, \quad (10)$$

which is an exact differential if the following conditions hold

$$\frac{\partial A}{\partial y} = \frac{\partial B}{\partial x}; \quad (11)$$

$$\frac{\partial f}{\partial x} = A, \quad (12)$$

$$\frac{\partial f}{\partial y} = B. \quad (13)$$

It follows from Eq. (10) that

$$f(x,y) = \int Adx + \int Bdy - \int\left(\frac{\partial}{\partial y}\left(\int Adx\right)\right)dy. \quad (14)$$

Comparing Eqs. (9) and (10), we get $A = \frac{1}{T}$ and $B = -\frac{\Phi}{T}$, where m and e will play the roles of x and y, respectively. The condition for an exact differential will become

$$\frac{\partial}{\partial e}\left(\frac{1}{T}\right) = \frac{\partial}{\partial m}\left(-\frac{\Phi}{T}\right). \tag{15}$$

Using Eq. (14), we can write the entropy for the BH in the integral form as

$$S(m,e) = \int \frac{1}{T} dm - \int \frac{\Phi}{T} de - \int \left(\frac{\partial}{\partial e}\left(\int \frac{1}{T} dm\right)\right) de. \tag{16}$$

When the thermodynamical quantities satisfy Eqs. (8) and (15), dS will be an exact differential. We can use the integral form in Eq. (16) to work out the semiclassical entropy for this black hole. It follows from Eq. (15) that the quantities T and Φ satisfy

$$\frac{\partial}{\partial e}\int \left(\frac{dm}{T}\right) = -\frac{\Phi}{T}. \tag{17}$$

Using this equation in Eq. (16), we obtain

$$S(m,e) = \int \frac{1}{T} dm. \tag{18}$$

This equation shows the semiclassical entropy of the charged BH, which is independent of Φ.

III. THERMODYNAMICAL QUANTITIES FOR THE BARDEEN MODEL

When particles escape, a BH loses a small amount of its energy (mass). The power emitted by a BH in the form of Hawking radiation can be estimated for a charged regular BH. The generalized form of BH solutions is given by

$$ds^2 = -F dt^2 + F^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \tag{19}$$

where $F = 1 - 2\frac{M(r)}{r}$. This metric can be reduced to a well-known BH for a special choice of $M(r)$. Ayon-Beato and Gracia [22] gave a physical interpretation of the Bardeen model [23] by showing that the charge associated

with it acts as a magnetic monopole charge. This is described by the metric with

$$M(r) = \frac{mr^3}{(r^2 + e^2)^{\frac{3}{2}}}. \tag{20}$$

Here, m and e stand for the mass and the monopole charge of a self-gravitating magnetic field of a non-linear electrodynamics source, respectively. This solution exhibits BH behavior for $e^2 \leq (16/27)m^2$ while it reduces to the Schwarzschild solution for $e = 0$.

The Bardeen regular black hole solution has a spherical event horizon at

$$r_+ = 2M(r_+), \tag{21}$$

where r_+ is the event horizon. Replacing the value of M, it follows that

$$1 - \frac{2mr_+^2}{(r_+^2 + e^2)^{\frac{3}{2}}} = 0, \tag{22}$$

whose roots are given in Ref.16 while its area [17] is

$$A = \int \sqrt{g_{\theta\theta} g_{\varphi\varphi}} d\theta\varphi = 4\pi r_+^2. \tag{23}$$

Let us write

$$F(r) = 1 - \frac{2mr^2}{(r^2 + e^2)^{\frac{3}{2}}}. \tag{24}$$

The event horizon related with temperature T_H [24, 25] is

$$T_H = \frac{\hbar F'(r)}{4\pi}|_{r=r_+} = \frac{\hbar m r_+ (r_+^2 - 2e^2)}{2\pi (r_+^2 + e^2)^{\frac{5}{2}}}, \tag{25}$$

where $F'(r)$ denotes the derivative of F with respect to r and $m = \frac{(r^2+e^2)^{\frac{3}{2}}}{2r^2}$. The electric potential is given by [26]

$$\Phi = \frac{\partial m}{\partial e}|_{r=r_+} = \frac{3e}{2r_+^2}(r_+^2 + e^2)^{\frac{1}{2}}. \tag{26}$$

With these thermodynamical quantities, the Bardeen regular BH satisfies the first law of thermodynamics, Eq. (8), and the condition in Eq. (15). Thus, the semiclassical entropy takes the form

$$S_0(m, e) = \int \frac{1}{T_H} dm = \frac{2\pi}{\hbar} \int \frac{(r_+^2 + e^2)^{\frac{5}{2}}}{m r_+ (r_+^2 - 2e^2)} dm. \tag{27}$$

To evaluate this integral, we use Eq. (22), which yields

$$dm = \frac{(r_+^2 + e^2)^{\frac{1}{2}}(r_+^2 - 2e^2)}{2r_+^3} dr_+. \tag{28}$$

Using this value in Eq. (27), we obtain semiclassical entropy

$$S_0 = \frac{2\pi}{\hbar} \int r_+ (1 + \frac{e^2}{r_+^2})^{\frac{3}{2}} dr_+ = \frac{1}{2\hbar} \int \frac{A}{r_+} (1 + \frac{e^2}{r_+^2})^{\frac{3}{2}} dr_+. \tag{29}$$

The integrated form of the above expression is

$$S_0 \;=\; 2\pi\hbar^{-1} \left((-\frac{e^2}{r} + \frac{r}{2})\sqrt{e^2 + r^2} + \frac{3}{2} e^2 \ln(r + \sqrt{e^2 + r^2}) \right). \tag{30}$$

Note that if we put charge $e = 0$ and $\hbar = 1$, we recover the Bekenstein-Hawking area law relating entropy and the horizon area, $S_0 = \frac{A}{4}$, as usually occurs in the Einstein gravity. In the following, we work out the corrected form by taking into account the quantum effects on the thermodynamical quantities (temperature and entropy) inside the event horizon of the charged regular BH.

1. Hawking Temperature Corrections

In this section, we find the correction to the Hawking temperature as a result of quantum effects for the Bardeen regular BH. The expression for the semiclassical Hawking temperature, Eq. (25), turns out to be

$$T_H = \frac{\hbar (r_+^2 - 2e^2)}{4\pi r_+ (r_+^2 + e^2)}. \tag{31}$$

The corrected temperature, Eq. (7), in terms of the horizon radius can be written as

$$T = T_H \left(1 - \frac{4\beta\hbar r^4}{(r^2 + e^2)^3}\right). \tag{32}$$

Using Eq. (32) in Eq. (31), we obtain the quantum correction of temperature T as

$$T = \frac{\hbar (1 - 2\frac{e^2}{r_+^2})}{4\pi r_+ (1 + \frac{e^2}{r_+^2})} \left(1 - \frac{4\hbar\beta}{r_+^2} (1 + \frac{e^2}{r_+^2})^{-3}\right). \tag{33}$$

For $e=0$, this reduces to the modified Hawking temperature of the Schwarzschild with

$$\beta = -\frac{1}{360\pi}\left(-N_0 - \frac{7}{4}N_{\frac{1}{2}} + 13N_1 + \frac{233}{4}N_{\frac{3}{2}} - 212N_2\right), \quad (34)$$

where N_s refers to the number of spin 's' fields [7].

2. Entropy Corrections

Here, we evaluate the quantum corrections to the entropy of the Bardeen charged regular BH. In terms of the horizon radius, the corrected form of entropy, Eq. (2), is given by

$$S(r,t) = S_0(r,t)\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right) \quad (35)$$

while the corrected form of the Hawking temperature can be written as

$$T = T_H\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right)^{-1}. \quad (36)$$

In the first law of thermodynamics, Eq. (9), we replace the temperature T by the corrected form of the temperature. Similarly, due to the corrected temperature, Eq. (15) takes the following form:

$$\frac{\partial}{\partial e}\left(\frac{1}{T_H}\right)\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right) = \frac{\partial}{\partial m}\left(-\frac{\Phi}{T_H}\right)\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right). \quad (37)$$

Thus, the entropy with the correction terms is given by

$$\begin{aligned} S(m,e) &= \int \frac{1}{T_H}\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right)dm - \int \frac{\Phi}{T_H}\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right)de \\ &- \int\left(\frac{\partial}{\partial e}\left(\int \frac{1}{T_H}\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right)dm\right)\right)de. \quad (38)\end{aligned}$$

This is the corrected and modified form of Eq. (16).

We can simplify these complicated integrals by employing the exactness criterion described above. As a result, Eq. (38) reduces to

$$S(m,e) = \int \frac{1}{T_H}\left(1 + \sum_i \frac{\alpha_i \hbar^i (4r_+^4)^i}{(r_+^2+e^2)^{3i}}\right)dm, \quad (39)$$

which can be written in expanded form as

$$\begin{aligned} S(m,e) &= \int \frac{1}{T_H}dm + \int \frac{\alpha_1 \hbar (4r_+^4)}{T_H (r_+^2+e^2)^3}dm + \int \frac{\alpha_2 \hbar^2 (4r_+^4)^2}{T_H (r_+^2+e^2)^6}dm \\ &+ \int \frac{\alpha_3 \hbar^3 (4r_+^4)^3}{T_H (r_+^2+e^2)^9}dm + ... \\ &= I_1 + I_2 + I_3 + I_4 + ..., \end{aligned} \tag{40}$$

where the first integral I_1 has been evaluated in Eq. (29) and $I_2, I_3, ...$ are quantum corrections. Thus,

$$I_2 = 2^3 \pi \alpha_1 \int \frac{r_+^2}{(r_+^2+e^2)^{\frac{3}{2}}} dr_+ \tag{41}$$

and

$$I_3 = 2^5 \pi \alpha_2 \hbar \int \frac{r_+^6}{(r_+^2+e^2)^{\frac{9}{2}}} dr_+. \tag{42}$$

In general, we can write

$$I_k = 2^{2k-1} \pi \alpha_{k-1} \hbar^{k-2} \int \frac{r_+^{4k-6}}{(r_+^2+e^2)^{\frac{6k-9}{2}}} dr_+, \quad k > 3. \tag{43}$$

Therefore, the entropy with quantum corrections is given by

$$\begin{aligned} S(m,e) &= 2\pi\hbar^{-1} \int \frac{(r_+^2+e^2)^{\frac{3}{2}}}{r_+^2} dr_+ + 2^3 \pi \alpha_1 \int \frac{r_+^2}{(r_+^2+e^2)^{\frac{3}{2}}} dr_+ \\ &+ \sum_{k>2} 2^{2k-1} \pi \alpha_{k-1} \hbar^{k-2} \int \frac{r_+^{4k-6}}{(r_+^2+e^2)^{\frac{6k-9}{2}}} dr_+. \end{aligned} \tag{44}$$

This gives the quantum correction to the entropy for a Bardeen charged BH.

For $e = 0$, Eq. (44) reduces to the entropy correction of the Schwarzschild black hole [7]; *i.e.*,

$$S = \frac{A}{4\hbar} + 4\pi\alpha_1 \ln A - \frac{64\pi^2 \hbar \alpha_2}{A} + ..., \tag{45}$$

where the area of the horizon, A, is given by Eq. (23). It is worth mentioning here that the first term of Eq. (45) is the semiclassical Bekenstein-Hawking area law, *i.e.*, $S_{BH} = \frac{A}{4\hbar}$; other terms are quantum corrections. Thus, S_{BH}

is modified by quantum effects. After the integrals are evaluated, Eq. (44) takes the form

$$\begin{aligned} S(m,e) &= 2\pi\hbar^{-1}\left((-\frac{e^2}{r}+\frac{r}{2})\sqrt{e^2+r^2}+\frac{3}{2}e^2\ln(r+\sqrt{e^2+r^2})\right) \\ &\quad +2^3\pi\alpha_1\left(\frac{-r}{\sqrt{e^2+r^2}}+\ln(r+\sqrt{e^2+r^2})\right) \\ &\quad +2^5\pi\hbar\alpha_2\left(\frac{r^7}{7e^2(e^2+r^2)^{\frac{7}{2}}}\right)+.... \end{aligned} \quad (46)$$

The entropy in Eq. (44) in terms of A is given as follows:

$$\begin{aligned} S(m,e) &= \pi\sqrt{4\pi}\hbar^{-1}\int\frac{(\frac{A}{4\pi}+e^2)^{\frac{3}{2}}}{A^{\frac{3}{2}}}dA+\frac{\alpha_1}{\sqrt{4\pi}}\int\frac{\sqrt{A}}{(\frac{A}{4\pi}+e^2)^{\frac{3}{2}}}dA \\ &+ \sum_{k>2}\frac{2^{2k-4}\hbar^{k-2}\alpha_{k-1}}{(4\pi)^{\frac{4k-7}{2}}}\int\frac{A^{\frac{4k-7}{2}}}{(\frac{A}{4\pi}+e^2)^{\frac{6k-9}{2}}}dA. \end{aligned} \quad (47)$$

When we take $e=0$, this equation leads to Eq.(45). Solving Eq. (47), we obtain

$$\begin{aligned} & S(m,e) = \frac{\pi\sqrt{4\pi}\hbar^{-1}}{8\sqrt{A}\pi^{\frac{3}{2}}} \\ & \left((A-8\pi e^2)\sqrt{A+4\pi e^2}+12\sqrt{A}e^2\pi\ln(\frac{\sqrt{\pi}}{2}(\sqrt{A}+\sqrt{A+4\pi e^2}))\right) \\ &+ \frac{\alpha_1 16\pi^{\frac{3}{2}}}{\sqrt{4\pi}}\left(\frac{-\sqrt{A}}{\sqrt{A+4e^2\pi}}+\ln(\frac{\sqrt{\pi}}{2}(\sqrt{A}+\sqrt{A+4\pi e^2}))\right) \\ &+ 4\hbar\alpha_2\left(\frac{8\pi A^{\frac{7}{2}}}{7e^2(A+4e^2\pi)^{\frac{7}{2}}}\right)+.... \end{aligned} \quad (48)$$

IV. OUTLOOK

Black holes are sites of immense gravitational attraction into which surrounding matter is drawn by gravitational forces. Classically, the gravitation is so powerful that nothing, not even electromagnetic radiation, can escape from the BH. Hawking defined a BH as a galactic monster that emits radiation due to quantum effects. The physical interpretation of this emission process indicates that vacuum fluctuations accelerate particle (positive

mass)-antiparticle (negative mass) pairs towards the event horizon of the BH. Hawking realized that a particle with positive mass has enough energy to escape from the BH while a particle with negative mass has no capability to escape from the BH and, hence, would fall in. This process of an in-falling negative energy particle eventually causes the mass of the BH to decrease. However, a particle that goes off to a distant observer would be observed as heat radiation. In fact, this process is a quantum tunneling effect, in which a particle-antiparticle pair will form from the vacuum, and one with positive energy will tunnel outside the event horizon with a complex action that appears as Hawking radiation. These radiations depend on the mass, angular momentum and charge of the BH. Negative energy particles are the main source of evaporation of a BH.

Using the above analysis, we have studied the quantum corrections to the temperature and the entropy of the Bardeen regular BH. The quantum correction to temperature is given by Eq. (33), which reduces to the Schwarzschild temperature [7] for $e = 0$. We write down the first law of thermodynamics for BHs as a differential of the entropy, including two parameters, mass and charge. Applying the condition for exactness of differentials, we evaluated the corrected entropy as a power series. For the charge to be zero, Eq. (44) yields the corrected entropy of the Schwarzschild BH (45). Here, the first term is the semiclassical value while the leading correction term is logarithmic. The other terms involve ascending powers of the inverse of the area [7]. The entropy correction in terms of the horizon area (47) also reduces to the Schwarzschild BH for the charge to be zero.

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